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AXISYMMETRIC DYNAMIC RESPONSE OF A CIRCULAR PLATE ON AN ELASTIC FOUNDATION

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1. INTRODUCTION

Circular plates on elastic foundations are used in footings and raft foundations of various structures. In addition, tubesheets used in various shell-and-tube type heat exchangers, can be modelled as circular plates on elastic foundations.

The static response of a circular plate on an elastic foundation is well studied [1–3, for example]. The axisymmetric dynamic response of a circular plate has been analytically studied [4, for example]. Free vibration of circular footing has been studied by some authors [5, for example].

This paper presents an analysis and numerical results for the axisymmetric free and forced vibration of a circular plate on an elastic foundation.

2. THEORY

The axisymmetric equilibrium of a circular plate represented by Poisson-Kirchhoff plate theory and resting on a Winkler medium [1] is represented in terms of the non-dimensional deflection, y and radius x as

$$D\left[\frac{\partial^4 y}{\partial x^4}+\frac{2}{x}\frac{\partial^3 y}{\partial x^3}-\frac{1}{x}\frac{\partial^2 y}{\partial x^2}+\frac{1}{x}\frac{\partial y}{\partial x}\right]+ka^4 y+\rho h a^4\frac{\partial^2 y}{\partial t^2}=pa^3. \tag{1}$$

where

$$y = w/a \qquad \text{and} \qquad x = r/a$$

where w is the deflection of the plate of radius a, thickness h, density ρ and flexural rigidity D. p is the uniformly distributed load per unit area of the plate. k is the spring constant for the foundation medium. The boundary conditions are:

For clamped edge,

$$y = 0 \quad \text{at} \quad x = 1, \quad \text{and} \quad \partial y/\partial x = 0 \quad \text{at} \quad x = 1. \qquad (2, 3)$$

For a simply supported edge,

$$y = 0 \quad \text{at} \quad x = 1, \qquad \partial^2 y/\partial x^2 + (v/x)\,\partial y/\partial x \qquad \text{at} \quad x = 1. \qquad (4, 5)$$

where v is Poisson's ratio of the plate material. The solution is required to be bounded everywhere.

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2.1. *Solution for free vibration*

A solution is sought in the following form:

$$y = R(x)\,\mathrm{e}^{\mathrm{i}\omega t}; \qquad \mathrm{i} = \sqrt{-1}.$$

Then equation (1) with the RHS set equal to zero yields

$$(\nabla^2 + \alpha^2)(\nabla^2 - \alpha^2)R = 0. \tag{6}$$

where,

$$\alpha^4 = (1/D)(\rho h a^4 \omega^2 - k a^4) \tag{7}$$

The frequency ω is non-dimensionalised as

$$\beta^4 = \rho h a^4 \omega^2 / D \tag{8}$$

From equation (7)

$$\beta^4 = \alpha^4 + K; \qquad K = k a^4 / D, \tag{9}$$

The solution to equation (6) is

$$R = A\mathrm{J}_0(\alpha x) + B\mathrm{Y}_0(\alpha x) + C\mathrm{I}_0(\alpha x) + D\mathrm{K}_0(\alpha x) \tag{10}$$

where the constants A, B, C and D are to be determined from the boundary conditions. J_0 and Y_0 are the zeroth order Bessel functions of the first and the second kind respectively. I_0 and K_0 are the zeroth order modified Bessel functions. From the boundedness of the solution at $x = 0$ it is required that $B = 0$ and $D = 0$, and hence the solution reduces to

$$R = A\mathrm{J}_0(\alpha x) + C\mathrm{I}_0(\alpha x).$$

For both simply supported and clamped boundaries $y = 0$, hence $R = 0$ at $x = 1$. Without any loss of generality,

$$R = \mathrm{I}_0(\alpha)\mathrm{J}_0(\alpha x) - \mathrm{J}_0(\alpha)\mathrm{I}_0(\alpha x). \tag{11}$$

2.1.1. Frequency equation for clamped edge. $\partial y/\partial x = 0$ i.e., $\mathrm{d}R/\mathrm{d}x = 0$ at $x = 1$. From equations (11), the characteristic equation is

$$\mathrm{J}_0(\alpha)\mathrm{I}_1(\alpha) + \mathrm{I}_0(\alpha)\mathrm{J}_1(\alpha) = 0. \tag{12}$$

From the solutions, α_i's of equation (12), the non-dimensional natural frequencies β_i's are obtained by using equation (9).

2.1.2. Frequency equation for simply supported edge. From equation (5) it follows $\mathrm{d}^2R/\mathrm{d}x^2 + (v/x)\,\mathrm{d}R/\mathrm{d}x = 0$ at $x = 1$. From equation (11), the characteristic equation for simply supported edge is obtained as

$$2\alpha\mathrm{I}_0(\alpha)\mathrm{J}_0(\alpha) + (v - 1)[\mathrm{I}_0(\alpha)\mathrm{J}_1(\alpha) + \mathrm{J}_0(\alpha)\mathrm{I}_1(\alpha)] = 0. \tag{13}$$

The usual recurrence relationships [6] have been used in deriving equations (12) and (13).

2.2. *Analysis of forced vibration*

A solution of the forced vibration equation (1) is sought in the form

$$y(x, t) = \sum R_j(x) g_j(t) \tag{14}$$

where $R_j(x)$ is the natural mode shape of the jth mode. Thus it is evident that the boundary conditions are automatically satisfied by equation (14). The unknown function of time $g_j(t)$ is to be determined. Substituting equation (14) in equation (1) one obtains by making use of equations (6) and (7),

$$\sum_j \left(\frac{\mathrm{d}^2 g_j}{\mathrm{d}t^2} + \omega^2 g_j\right) R_j = \frac{p}{\rho h a} \tag{15}$$

Each side of equation (15) is multiplied by $xR_i(x)$ and integrated between the limits 0 and 1.

$$\int_0^1 \sum_j \left(\frac{\mathrm{d}^2 g_j}{\mathrm{d}t^2} + \omega^2 g_j\right) x R_i R_j \,\mathrm{d}x = \int_0^1 \frac{p}{\rho h a} x R_i \,\mathrm{d}x, \qquad N_{ij} = \int_0^1 x R_i R_j \,\mathrm{d}x \tag{16}$$

N_{ij} has been evaluated making use of the orthogonality of the natural modes [4, 6]. For the plate with a clamped edge,

$$N_{ij} = \{\tfrac{1}{2}[\mathrm{I}_0^2(\alpha_i)\mathrm{J}_1^2(\alpha_i) - \mathrm{J}_0^2(\alpha_i)\mathrm{I}_1^2(\alpha_i)] + \mathrm{I}_0^2(\alpha_i)\mathrm{J}_0^2(\alpha_i)\}\delta_{ij}. \tag{17}$$

For simply supported edge,

$$N_{ij} = \{\tfrac{1}{2}[\mathrm{I}_0^2(\alpha_i)\mathrm{J}_1^2(\alpha_i) - \mathrm{J}_0^2(\alpha_i)\mathrm{I}_1^2(\alpha_i)] - [(1+\nu)/(1-\nu]\mathrm{I}_0^2(\alpha_i)\mathrm{J}_0^2(\alpha_i)\}\delta_{ij}, \tag{18}$$

where δ_{ij} is the Kronecker delta. Hence equation (16) reduces to

$$\frac{\mathrm{d}^2 g_j}{\mathrm{d}t^2} + \omega_j^2 g_j = \frac{1}{N_{jj}\rho a h}\int_0^1 p(x, t) x R_j(x)\,\mathrm{d}x. \tag{19}$$

For uniform pressure, $p = p_0 f_1(t)$ the integral on the right side of equation (19) is

$$= p_0 f_2(\alpha_j) f_1(t), \tag{20}$$

$$f_2(\alpha_j) = [\mathrm{I}_0(\alpha_j)\mathrm{J}_1(\alpha_j) - \mathrm{J}_0(\alpha_j)\mathrm{I}_1(\alpha_j)]/\alpha_j. \tag{21}$$

So equation (19) is rewritten as

$$\mathrm{d}^2 g_j/\mathrm{d}t^2 + \omega_j^2 g_j = (p_0 f_2(\alpha_j)/N_{jj}\rho a h) f_1(t). \tag{22}$$

For zero initial conditions, the solution to equation (22) is

$$g_j = \frac{p_0 f_2(\alpha_j)}{\omega_j N_{jj}\rho a h}\int_0^t f_1(t-\tau)\sin(\omega_j\tau)\mathrm{d}\tau. \tag{23}$$

Non-dimensional time θ is defined as

$$\theta = \omega_1 t \qquad \text{and} \qquad \gamma_j = \omega_j/\omega_1 = \beta_j^2/\beta_1^2. \tag{24, 25}$$

For step loading i.e., $f_1(t) = 1$,

$$g_j = (p_0 a^3/D)(f_2(\alpha_j)/\beta_j^4 N_{jj})[1 - \cos(\gamma_j\theta)]. \tag{26}$$

For sinusoidal loading,

$$f_1(t) = \sin(\Omega t) = \sin(f\omega_1 t) = \sin(f\theta); \qquad (\Omega = f\omega_1), \tag{27}$$

$$g_j = \frac{p_0 a^3 f_2(\alpha_j)}{D\beta_j^4 N_{jj}(f^2/\gamma_j^2 - 1)}\left[\frac{f}{\gamma_j}\sin(\gamma_j\theta) - \sin(f\theta)\right]; \qquad (\Omega \neq \omega_j), \tag{28}$$

$$g_j = \frac{p_0 a^3 f_2(\alpha_j)}{D2f\beta_1^2\beta_j^2 N_{jj}}[\sin(f\theta) - f\theta\sin(f\theta)]; \qquad (\Omega = \omega_j). \tag{29}$$

From equations (26)–(29) it is seen that g_j can be written in the form $g_j = (p_0 a^3/D) f_3(\alpha_j) f_0(\theta)$, where $f_3(\alpha_j)$ and $f_0(\theta)$ depend on the type of loading. Pressure loading having other types of dependence on time may be represented by a sine series and the above results can be used. The deflection and the moments are normalised as follows.

$$w_n = w/(p_0 a^4/D) = ya/(p_0 a^4/D)$$

$$= \sum_j [\mathrm{I}_0(\alpha_j)\mathrm{J}_0(\alpha_j x) - \mathrm{J}_0(\alpha_j)\mathrm{I}_0(\alpha_j x)] f_3(\alpha_j) f_0(\theta), \tag{30}$$

$$M_r = -D\left[\sum_j \frac{\mathrm{d}^2 R_j}{\mathrm{d}x^2} + \frac{v}{x}\frac{\mathrm{d}R_j}{\mathrm{d}x}\Bigg) g_j\right]\Bigg/ a,$$

$$M_{r,n} = M_r/p_0 a^2 = \sum_j \left(\frac{\mathrm{d}^2 R_j}{\mathrm{d}x^2} + \frac{v}{x}\frac{\mathrm{d}R_j}{\mathrm{d}x}\right) f_3(\alpha_j) f_0(\theta), \tag{31}$$

$$M_{t,n} = \frac{M_t}{p_0 a^2} = \sum_j \left(\dot{v}\frac{\mathrm{d}^2 R_j}{\mathrm{d}x^2} + \frac{1}{x}\frac{\mathrm{d}R_j}{\mathrm{d}x}\right) f_3(\alpha_j) f_0(\theta). \tag{32}$$

At $x = 0$, $M_r = M_t$.

3. NUMERICAL RESULTS

3.1. *Vibration of a plate with clamped edge*

3.1.1. Free vibration. Equation (12) is solved for the eigenvalues α's from which the non-dimensional natural frequencies β's are evaluated by equation (9). In this case, β is found to be dependent only on the non-dimensional parameter $K = ka^4/D$. The values of β for the first ten modes are presented in Table 1 for five different values of K. It is seen that β increases with K. This is due to the increase in the overall stiffness of the plate–foundation system. The solution of the forced vibration equation converges quite rapidly. The results for $K = 0$ i.e., no foundation are in agreement with those presented in [4]. The analytical results are a special case of those presented in [5].

3.1.2. Forced vibration. As a typical transient response to a uniformly distributed pressure varying as a step function of time the normalised radial moment at the centre

Table 1

Non-dimensional natural frequencies (β) of a plate with clamped edge

	K				
Mode no.	0·000E + 00	0·100E + 02	0·500E + 02	0·100E + 03	0·200E + 03
1	0·319E + 01	0·327E + 01	0·352E + 01	0·378E + 01	0·418E + 01
2	0·630E + 01	0·631E + 01	0·635E + 01	0·640E + 01	0·650E + 01
3	0·944E + 01	0·944E + 01	0·945E + 01	0·947E + 01	0·950E + 01
4	0·126E + 02	0·126E + 02	0·126E + 02	0·126E + 02	0·126E + 02
5	0·157E + 02	0·157E + 02	0·157E + 02	0·157E + 02	0·157E + 02
6	0·189E + 02	0·189E + 02	0·189E + 02	0·189E + 02	0·189E + 02
7	0·220E + 02	0·220E + 02	0·220E + 02	0·220E + 02	0·220E + 02
8	0·251E + 02	0·251E + 02	0·251E + 02	0·251E + 02	0·251E + 02
9	0·283E + 02	0·283E + 02	0·283E + 02	0·283E + 02	0·283E + 02
10	0·314E + 02	0·314E + 02	0·314E + 02	0·314E + 02	0·314E + 02

is presented in Figure 1 for the case $K = 0$ and $v = 0{\cdot}2$. It is seen that the results oscillate about the static solution, $M_{r,n} = (1 + v)/16 = 0{\cdot}075$ [1]. This oscillation being undamped, the maximum amplitude remains unaltered. It is seen that the maximum dynamic radial moment is a significantly amplified version of the corresponding static value.

The maximum values of the normalised deflection and radial and the tangential moments due to the step loading for various values of K and $v = 0{\cdot}2$ are presented in Table 2.

The response to sinusoidal loading is studied next. It is found that the peak deflection and the peak moments depend on the frequency ratio f (see equation (27)) and K. The peak moments depend on Poisson's ratio v as well. The peak deflection and the peak

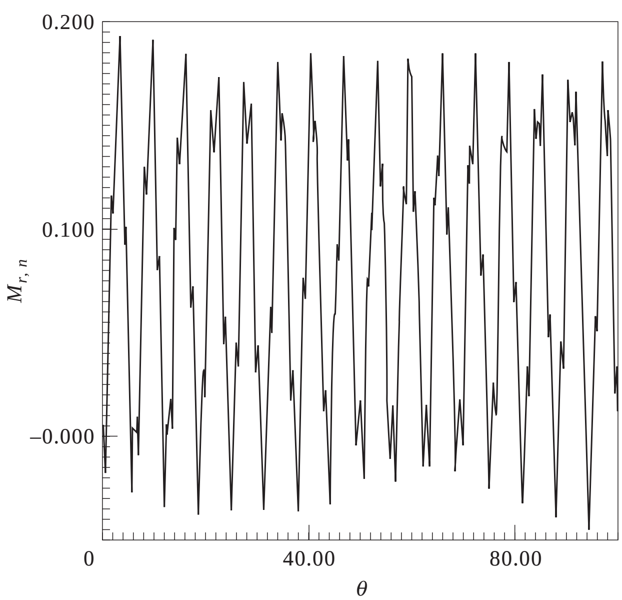

Figure 1. Variation of $M_{r,n}$ with θ; clamped edge; $x = 0$; step loading; $K = 0$; $v = 0{\cdot}20$.

TABLE 2

Normalised values of peak deflection and moments at the centre of the plate for step function loading (clamped edge)

	$\nu = 0{\cdot}20$				
K	0·000E + 00	0·100E + 02	0·500E + 02	0.100E + 03	0·200E + 03
w_n	0·329E − 01	0·300E − 01	0·222E − 01	0·168E − 01	0·113E − 01
$M_{r,n}$	0·193E + 00	0·183E + 00	0·132E + 00	0·105E + 00	0·766E − 01

tangential moment occur at the centre. However, the location of the peak radial moment varies with f and K.

The magnitudes of peak w_n and $M_{r,n}$ respectively as a function of f are presented in Table 3 for various values of K. The peak deflection is independent of the value of ν. The results for the moments are presented for $\nu = 0{\cdot}2$. Figure 2 shows the peak radial moment as a function of f for $K = 50$ and $\nu = 0{\cdot}2$.

3.2. *Vibration of a plate with simply supported edge*

3.2.1. Free vibration. Equation (13) is solved for the eigenvalues α's from which the

TABLE 3

Peak amplitude under sinusoidal loading (clamped edge)

(A) *Normalised deflection* (w_n)

	K				
f	0	10	50	100	200
0·200E + 00	0·171E − 01	0·156E − 01	0·114E − 01	0·848E − 02	0·557E − 02
0·400E + 00	0·259E − 01	0·236E − 01	0·173E − 01	0·129E − 01	0·847E − 02
0·600E + 00	0·401E − 01	0·366E − 01	0·270E − 01	0·202E − 01	0·135E − 01
0·800E + 00	0·798E − 01	0·727E − 01	0·537E − 01	0·405E − 01	0·267E − 01
0·900E + 00	0·162E + 00	0·148E + 00	0·109E + 00	0·825E − 01	0·550E − 01
0·950E + 00	0·325E + 00	0·297E + 00	0·219E + 00	0·165E + 00	0·111E + 00
0·105E + 01	0·327E + 00	0·298E + 00	0·221E + 00	0·167E + 00	0·112E + 00
0·110E + 00	0·164E + 00	0·149E + 00	0·111E + 00	0·839E − 01	0·562E − 01
0·120E + 01	0·818E − 01	0·747E − 01	0·555E − 01	0·424E − 01	0·285E − 01
0·150E + 01	0·322E − 01	0·295E − 01	0·221E − 01	0·172E − 01	0·124E − 01
(B) *Normalised radial moment* ($M_{r,n}$) ($\nu = 0{\cdot}20$)					
0·200E + 00	0·131E + 00	0·121E + 00	0·953E − 01	0·771E − 01	0·583E − 01
0·400E + 00	0·189E + 00	0·175E + 00	0·136E + 00	0·108E + 00	0·801E − 01
0·600E + 00	0·279E + 00	0·257E + 00	0·197E + 00	0·155E + 00	0·113E + 00
0·800E + 00	0·524E + 00	0·481E + 00	0·364E + 00	0·280E + 00	0·198E + 00
0·900E + 00	0·103E + 01	0·943E + 00	0·708E + 00	0·541E + 00	0·367E + 00
0·950E + 00	0·204E + 01	0·187E + 01	0·139E + 01	0·106E + 01	0·716E + 00
0·105E + 01	0·200E + 01	0·182E + 01	0·133E + 01	0·101E + 01	0·662E + 00
0·110E + 00	0·982E + 00	0·897E + 00	0·657E + 00	0·493E + 00	0·321E + 00
0·120E + 01	0·478E + 00	0·435E + 00	0·319E + 00	0·251E + 00	0·172E + 00
0·150E + 01	0·196E + 00	0·182E + 00	0·143E + 00	0·118E + 00	0·978E − 01

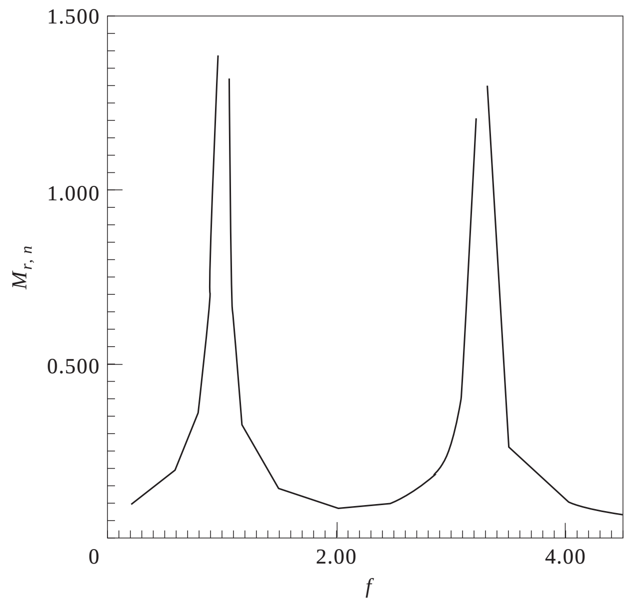

Figure 2. Variation of maximum $M_{r,n}$; clamped edge; $K = 50$; $v = 0{\cdot}20$.

non-dimensional natural frequencies β's are evaluated: α and β are dependent on both v and K. Table 4 presents the non-dimensional natural frequencies for different values of v and K. It is observed that β is practically insensitive to K beyond the first three modes.

TABLE 4

Non-dimensional natural frequencies (β) of a plate with simply supported edge

		K				
Mode no.	v	0·000E + 00	0·100E + 02	0·500E + 02	0·100E + 03	0·200E + 03
1	0·15	0·217E + 01	0·238E + 01	0·291E + 01	0·332E + 01	0·386E + 01
2		0·554E + 01	0·545E + 01	0·551E + 01	0·559E + 01	0·572E + 01
3		0·860E + 01	0·861E + 01	0·862E + 01	0·864E + 01	0·868E + 01
4		0·118E + 02	0·118E + 02	0·118E + 02	0·118E + 02	0·118E + 02
5		0·149E + 02	0·149E + 02	0·149E + 02	0·149E + 02	0·149E + 02
6		0·180E + 02	0·180E + 02	0·180E + 02	0·181E + 02	0·181E + 02
7		0·212E + 02	0·212E + 02	0·212E + 02	0·212E + 02	0·212E + 02
8		0·243E + 02	0·243E + 02	0·243E + 02	0·243E + 02	0·243E + 02
9		0·275E + 02	0·275E + 02	0·275E + 02	0·275E + 02	0·275E + 02
10		0·306E + 02	0·306E + 02	0·306E + 02	0·306E + 02	0·306E + 02
1	0·20	0·219E + 01	0·239E + 01	0·292E + 01	0·333E + 01	0·386E + 01
2		0·544E + 01	0·546E + 01	0·552E + 01	0·559E + 01	0·573E + 01
3		0·861E + 01	0·861E + 01	0·863E + 01	0·864E + 01	0·868E + 01
4		0·118E + 02	0·118E + 02	0·118E + 02	0·118E + 02	0·118E + 02
1	0·25	0·220E + 01	0·241E + 01	0·293E + 01	0·333E + 01	0·387E + 01
2		0·545E + 01	0·546E + 01	0·552E + 01	0·560E + 01	0·573E + 01
3		0·861E + 01	0·861E + 01	0·863E + 01	0·865E + 01	0·869E + 01
4		0·118E + 02	0·118E + 02	0·118E + 02	0·118E + 02	0·118E + 02

TABLE 5

Normalised values of peak deflection and moments at the centre of the plate for step function loading (simply supported edge)

	ν	K: 0·000E + 00	0·100E + 02	0·500E + 02	0·100E + 03	0·200E + 03
w_n	–	0·142E + 00	0·981E − 01	0·436E − 01	0·259E − 01	0·143E − 01
$M_{r,n}$	0·15	0·439E + 00	0·301E + 00	0·141E + 00	0·872E − 01	0·553E − 01
w_n	–	0·138E + 00	0·958E − 01	0·433E − 01	0·257E − 01	0·142E − 01
$M_{r,n}$	0·20	0·438E + 00	0·309E + 00	0·145E + 00	0·894E − 01	0·564E − 01
w_n	–	0·134E + 00	0·938E − 01	0·429E − 01	0·256E − 01	0·142E − 01
$M_{r,n}$	0·25	0·458E + 00	0·321E + 00	0·153E + 00	0·965E − 01	0·602E − 01

Hence detailed results are presented only for $\nu = 0{\cdot}15$. The results for $K = 0$ are in agreement with those presented in [4].

3.2.2. Forced vibration. The maximum values of the normalised deflection and radial and tangential moments due to a step loading for various values of ν and K are

TABLE 6

Peak amplitude under sinusoidal loading (simply supported edge)

(A) *Normalised deflection* ($\nu = 0{\cdot}2$)

f	K: 0	10	50	100	200
0·200E + 00	0·736E − 01	0·509E − 01	0·225E − 01	0·131E − 01	0·685E − 02
0·400E + 00	0·111E + 00	0·767E − 01	0·342E − 01	0·200E − 01	0·106E − 01
0·600E + 00	0·171E + 00	0·119E + 00	0·531E − 01	0·309E − 01	0·170E − 01
0·800E + 00	0·338E + 00	0·235E + 00	0·105E + 00	0·620E − 01	0·340E − 01
0·900E + 00	0·684E + 00	0·476E + 00	0·214E + 00	0·127E + 00	0·696E − 01
0·950E + 00	0·137E + 01	0·955E + 00	0·430E + 00	0·255E + 00	0·140E + 00
0·105E + 01	0·137E + 01	0·957E + 00	0·432E + 00	0·256E + 00	0·142E + 00
0·110E + 01	0·687E + 00	0·478E + 00	0·216E + 00	0·128E + 00	0·716E − 01
0·120E + 01	0·341E + 00	0·238E + 00	0·108E + 00	0·647E − 01	0·361E − 01
0·150E + 01	0·132E + 00	0·923E − 01	0·427E − 01	0·264E − 01	0·161E − 01
0·200E + 01	0·610E − 01	0·430E − 01	0·208E − 01	0·143E − 01	0·160E − 01

(B) *Normalised radial moment* ($M_{r,n}$) ($\nu = 0{\cdot}20$)

f	K: 0	10	50	100	200
0·200E + 00	0·221E + 00	0·150E + 00	0·616E − 01	0·328E − 01	0·128E − 01
0·400E + 00	0·337E + 00	0·228E + 00	0·975E − 01	0·543E − 01	0·245E − 01
0·600E + 00	0·526E + 00	0·361E + 00	0·157E + 00	0·852E − 01	0·470E − 01
0·800E + 00	0·105E + 01	0·726E + 00	0·317E + 00	0·183E + 00	0·996E − 01
0·900E + 00	0·214E + 01	0·148E + 01	0·658E + 00	0·391E + 00	0·211E + 00
0·950E + 00	0·430E + 01	0·298E + 01	0·133E + 01	0·790E + 00	0·430E + 00
0·105E + 01	0·433E + 01	0·302E + 01	0·137E + 01	0·820E + 00	0·467E + 00
0·110E + 01	0·217E + 01	0·152E + 01	0·699E + 00	0·417E + 00	0·242E + 00
0·120E + 01	0·109E + 01	0·756E + 00	0·358E + 00	0·221E + 00	0·132E + 00
0·150E + 01	0·432E + 00	0·308E + 00	0·156E + 00	0·110E + 00	0·876E − 01
0·200E + 01	0·211E + 00	0·157E + 00	0·954E − 01	0·923E − 01	0·197E + 00

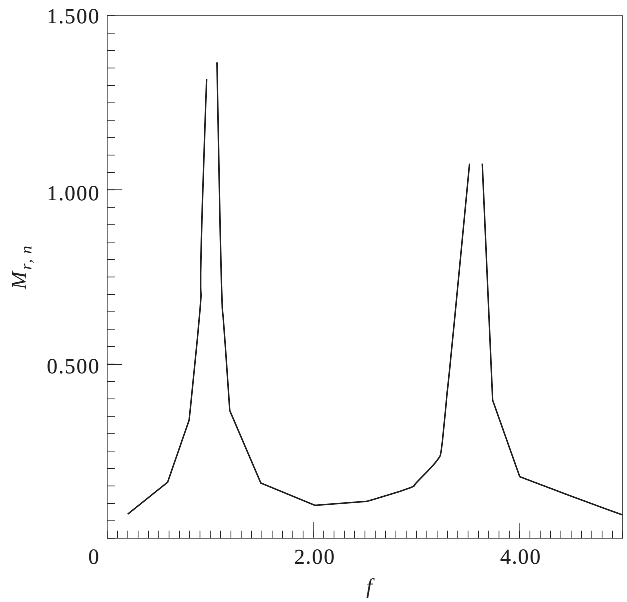

Figure 3. Variation of peak $M_{r,n}$ with f; simply supported edge; $K = 50$; $v = 0{\cdot}20$.

presented in Table 5. Unlike the case of the clamped edge, the deflection is also dependent on v.

The values of normalised peak deflection and moment at $x = 0$ for sinusoidal loading are presented in Table 6 for various values of K. Expectedly, the peak responses reduce with increase in K. Figure 3 shows the peak $M_{r,n}$ as a function of f for $K = 50$ and $v = 0{\cdot}2$.

4. CONCLUSIONS

Analytical results have been presented for the free and forced vibration of a circular plate on an elastic foundation. The various numerical results presented should be useful for practical applications.

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